Mathematica 11.3 Integration Test Results

on the problems in "4 Trig functions\4.6 Cosecant"

Test results for the 70 problems in "4.6.0 (a csc)^m (b trg)^n.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[a+b x] \, dx$$

Optimal (type 3, 12 leaves, 1 step):

$$-\frac{\text{ArcTanh}[\text{Cos}[a+b x]]}{b}$$

Result (type 3, 38 leaves):

$$-\frac{\text{Log}\left[\text{Cos}\left[\frac{a}{2}+\frac{b x}{2}\right]\right]}{b}+\frac{\text{Log}\left[\text{Sin}\left[\frac{a}{2}+\frac{b x}{2}\right]\right]}{b}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[a+b x]^3 \, dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$-\frac{\text{ArcTanh}[\text{Cos}[a+b x]]}{2 b}-\frac{\text{Cot}[a+b x] \, \text{Csc}[a+b x]}{2 b}$$

Result (type 3, 75 leaves):

$$-\frac{\text{Csc}\left[\frac{1}{2}(a+b x)\right]^2}{8 b}-\frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}(a+b x)\right]\right]}{2 b}+\frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}(a+b x)\right]\right]}{2 b}+\frac{\text{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{8 b}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[a+b x]^5 \, dx$$

Optimal (type 3, 55 leaves, 3 steps):

$$-\frac{3 \text{ArcTanh}[\text{Cos}[a+b x]]}{8 b}-\frac{3 \text{Cot}[a+b x] \text{Csc}[a+b x]}{8 b}-\frac{\text{Cot}[a+b x] \text{Csc}[a+b x]^3}{4 b}$$

Result (type 3, 113 leaves):

$$-\frac{3 \text{Csc}\left[\frac{1}{2} (a+b x)\right]^2}{32 b}-\frac{\text{Csc}\left[\frac{1}{2} (a+b x)\right]^4}{64 b}-\frac{3 \text{Log}\left[\text{Cos}\left[\frac{1}{2} (a+b x)\right]\right]}{8 b}+\frac{3 \text{Log}\left[\text{Sin}\left[\frac{1}{2} (a+b x)\right]\right]}{8 b}+\frac{3 \text{Sec}\left[\frac{1}{2} (a+b x)\right]^2}{32 b}+\frac{\text{Sec}\left[\frac{1}{2} (a+b x)\right]^4}{64 b}$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \left(\text{Csc}[x]^2\right)^{3/2} dx$$

Optimal (type 3, 22 leaves, 3 steps):

$$-\frac{1}{2} \text{ArcSinh}[\text{Cot}[x]]-\frac{1}{2} \text{Cot}[x] \sqrt{\text{Csc}[x]^2}$$

Result (type 3, 51 leaves):

$$\frac{1}{8} \sqrt{\text{Csc}[x]^2} \left(-\text{Csc}\left[\frac{x}{2}\right]^2-4 \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+4 \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\text{Sec}\left[\frac{x}{2}\right]^2\right) \text{Sin}[x]$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\text{Csc}[x]^2} \, dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$-\text{ArcSinh}[\text{Cot}[x]]$$

Result (type 3, 28 leaves):

$$\sqrt{\text{Csc}[x]^2} \left(-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]+\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]\right) \text{Sin}[x]$$

Test results for the 84 problems in "4.6.11 (e x)^m (a+b csc(c+d x^n))^p.m"

Problem 5: Result more than twice size of optimal antiderivative.

$$\int x \left(a + b \operatorname{Csc}\left[c + d x^2\right]\right) dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$\frac{a x^2}{2} - \frac{b \operatorname{ArcTanh}\left[\operatorname{Cos}\left[c + d x^2\right]\right]}{2 d}$$

Result (type 3, 57 leaves):

$$\frac{a x^2}{2} - \frac{b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{c}{2} + \frac{d x^2}{2}\right]\right]}{2 d} + \frac{b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{c}{2} + \frac{d x^2}{2}\right]\right]}{2 d}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a + b \operatorname{Csc}\left[c + d x^2\right]\right)^2 dx$$

Optimal (type 4, 125 leaves, 10 steps):

$$\frac{a^2 x^4}{4} - \frac{2 a b x^2 \operatorname{ArcTanh}\left[e^{i \left(c+d x^2\right)}\right]}{d} - \frac{b^2 x^2 \operatorname{Cot}\left[c + d x^2\right]}{2 d} + \frac{b^2 \operatorname{Log}\left[\operatorname{Sin}\left[c + d x^2\right]\right]}{2 d^2} + \frac{i a b \operatorname{PolyLog}\left[2, -e^{i \left(c+d x^2\right)}\right]}{d^2} - \frac{i a b \operatorname{PolyLog}\left[2, e^{i \left(c+d x^2\right)}\right]}{d^2}$$

Result (type 4, 590 leaves):

$$\frac{b^2 x^2 \mathrm{Cot}[c] \left(a+b\,\mathrm{Csc}\left[c+d x^2\right]\right)^2 \mathrm{Sin}\left[c+d x^2\right]^2}{2 d \left(b+a\,\mathrm{Sin}\left[c+d x^2\right]\right)^2}+\frac{x^2 \mathrm{Csc}\left[\frac{c}{2}\right] \left(a+b\,\mathrm{Csc}\left[c+d x^2\right]\right)^2 \mathrm{Sec}\left[\frac{c}{2}\right] \left(-2 b^2 \mathrm{Cos}[c]+a^2 d x^2 \mathrm{Sin}[c]\right) \mathrm{Sin}\left[c+d x^2\right]^2}{8 d \left(b+a\,\mathrm{Sin}\left[c+d x^2\right]\right)^2}+$$

$$\left(b^2 \mathrm{Csc}[c] \left(a+b\,\mathrm{Csc}\left[c+d x^2\right]\right)^2 \left(-d x^2 \mathrm{Cos}[c]+\mathrm{Log}\left[\mathrm{Cos}\left[d x^2\right] \mathrm{Sin}[c]+\mathrm{Cos}[c] \mathrm{Sin}\left[d x^2\right]\right] \mathrm{Sin}[c]\right) \mathrm{Sin}\left[c+d x^2\right]^2\right) \Big/$$

$$\left(2 d^2 \left(\mathrm{Cos}[c]^2+\mathrm{Sin}[c]^2\right) \left(b+a\,\mathrm{Sin}\left[c+d x^2\right]\right)^2\right)+\frac{b^2 x^2 \mathrm{Csc}\left[\frac{c}{2}\right] \mathrm{Csc}\left[\frac{c}{2}+\frac{d x^2}{2}\right] \left(a+b\,\mathrm{Csc}\left[c+d x^2\right]\right)^2 \mathrm{Sin}\left[\frac{d x^2}{2}\right] \mathrm{Sin}\left[c+d x^2\right]^2}{4 d \left(b+a\,\mathrm{Sin}\left[c+d x^2\right]\right)^2}+$$

$$\frac{b^2 x^2 \left(a+b\,\mathrm{Csc}\left[c+d x^2\right]\right)^2 \mathrm{Sec}\left[\frac{c}{2}\right] \mathrm{Sec}\left[\frac{c}{2}+\frac{d x^2}{2}\right] \mathrm{Sin}\left[\frac{d x^2}{2}\right] \mathrm{Sin}\left[c+d x^2\right]^2}{4 d \left(b+a\,\mathrm{Sin}\left[c+d x^2\right]\right)^2}+\frac{1}{d^2 \left(b+a\,\mathrm{Sin}\left[c+d x^2\right]\right)^2}$$

$$a b \left(a+b\,\mathrm{Csc}\left[c+d x^2\right]\right)^2 \mathrm{Sin}\left[c+d x^2\right]^2 \left(-\frac{2\,\mathrm{ArcTan}[\mathrm{Tan}[c]]\,\mathrm{ArcTanh}\left[\frac{-\mathrm{Cos}[c]+\mathrm{Sin}[c]\,\mathrm{Tan}\left[\frac{d x^2}{2}\right]}{\sqrt{\mathrm{Cos}[c]^2+\mathrm{Sin}[c]^2}}\right]}{\sqrt{\mathrm{Cos}[c]^2+\mathrm{Sin}[c]^2}}+\right.$$

$$\frac{1}{\sqrt{1+\mathrm{Tan}[c]^2}}\left(\left(d x^2+\mathrm{ArcTan}[\mathrm{Tan}[c]]\right) \left(\mathrm{Log}\left[1-e^{i \left(d x^2+\mathrm{ArcTan}[\mathrm{Tan}[c]]\right)}\right]-\mathrm{Log}\left[1+e^{i \left(d x^2+\mathrm{ArcTan}[\mathrm{Tan}[c]]\right)}\right]\right)+\right.$$

$$\left.\left. i \left(\mathrm{PolyLog}\left[2,-e^{i \left(d x^2+\mathrm{ArcTan}[\mathrm{Tan}[c]]\right)}\right]-\mathrm{PolyLog}\left[2,e^{i \left(d x^2+\mathrm{ArcTan}[\mathrm{Tan}[c]]\right)}\right]\right)\right) \mathrm{Sec}[c]\right)$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{a+b\,\mathrm{Csc}\left[c+d x^2\right]}\,dx$$

Optimal (type 4, 271 leaves, 11 steps):

$$\frac{x^4}{4 a}+\frac{i b x^2 \mathrm{Log}\left[1-\frac{i a e^{i \left(c+d x^2\right)}}{b-\sqrt{-a^2+b^2}}\right]}{2 a \sqrt{-a^2+b^2}\, d}-\frac{i b x^2 \mathrm{Log}\left[1-\frac{i a e^{i \left(c+d x^2\right)}}{b+\sqrt{-a^2+b^2}}\right]}{2 a \sqrt{-a^2+b^2}\, d}+\frac{b\,\mathrm{PolyLog}\left[2,\frac{i a e^{i \left(c+d x^2\right)}}{b-\sqrt{-a^2+b^2}}\right]}{2 a \sqrt{-a^2+b^2}\, d^2}-\frac{b\,\mathrm{PolyLog}\left[2,\frac{i a e^{i \left(c+d x^2\right)}}{b+\sqrt{-a^2+b^2}}\right]}{2 a \sqrt{-a^2+b^2}\, d^2}$$

Result (type 4, 1104 leaves):

$$\frac{x^4 \operatorname{Csc}[c+d x^2] (b+a \operatorname{Sin}[c+d x^2])}{4 a (a+b \operatorname{Csc}[c+d x^2])} -$$

$$\frac{1}{2 a d^2 (a+b \operatorname{Csc}[c+d x^2])} b \operatorname{Csc}[c+d x^2] \left(\frac{\pi \operatorname{ArcTan}\left[\frac{a+b \operatorname{Tan}\left[\frac{1}{2}(c+d x^2)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}} + \frac{1}{\sqrt{a^2-b^2}} \left(2 \left(-c+\frac{\pi}{2}-d x^2\right) \operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] -\right.\right.$$

$$2 \left(-c+\operatorname{ArcCos}\left[-\frac{b}{a}\right]\right) \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] + \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] - 2 i\right.$$

$$\left.\left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right) \operatorname{Log}\left[\frac{\sqrt{a^2-b^2}\, e^{-\frac{1}{2} i \left(-c+\frac{\pi}{2}-d x^2\right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Sin}[c+d x^2]}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{(a+b) \operatorname{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right] - \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\operatorname{Log}\left[\frac{\sqrt{a^2-b^2}\, e^{\frac{1}{2} i \left(-c+\frac{\pi}{2}-d x^2\right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \operatorname{Sin}[c+d x^2]}}\right] - \left(\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\operatorname{Log}\left[1-\frac{\left(b-i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}\right] +$$

$$\left(-\operatorname{ArcCos}\left[-\frac{b}{a}\right] + 2 i \operatorname{ArcTanh}\left[\frac{(a-b) \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right) \operatorname{Log}\left[1-\frac{\left(b+i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}\right] +$$

$$i \left(\operatorname{PolyLog}\left[2, \frac{\left(b-i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}\right] -\right.$$

$$\left.\left.\left.\operatorname{PolyLog}\left[2, \frac{\left(b+i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \operatorname{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^2\right)\right]\right)}\right]\right)\right)\right) (b+a \operatorname{Sin}[c+d x^2])$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Csc}\left[\sqrt{x}\right]^3}{\sqrt{x}}\, dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$-\mathrm{ArcTanh}\left[\mathrm{Cos}\left[\sqrt{x}\right]\right] - \mathrm{Cot}\left[\sqrt{x}\right]\mathrm{Csc}\left[\sqrt{x}\right]$$

Result (type 3, 57 leaves):

$$-\frac{1}{4}\mathrm{Csc}\left[\frac{\sqrt{x}}{2}\right]^2 - \mathrm{Log}\left[\mathrm{Cos}\left[\frac{\sqrt{x}}{2}\right]\right] + \mathrm{Log}\left[\mathrm{Sin}\left[\frac{\sqrt{x}}{2}\right]\right] + \frac{1}{4}\mathrm{Sec}\left[\frac{\sqrt{x}}{2}\right]^2$$

Problem 75: Unable to integrate problem.

$$\int (e\,x)^{-1+3n}\left(a + b\,\mathrm{Csc}\left[c + d\,x^n\right]\right) dx$$

Optimal (type 4, 221 leaves, 11 steps):

$$\frac{a\,(e\,x)^{3n}}{3\,e\,n} - \frac{2\,b\,x^{-n}\,(e\,x)^{3n}\,\mathrm{ArcTanh}\left[e^{i\,(c+d\,x^n)}\right]}{d\,e\,n} + \frac{2\,i\,b\,x^{-2n}\,(e\,x)^{3n}\,\mathrm{PolyLog}\left[2, -e^{i\,(c+d\,x^n)}\right]}{d^2\,e\,n} - \frac{2\,i\,b\,x^{-2n}\,(e\,x)^{3n}\,\mathrm{PolyLog}\left[2, e^{i\,(c+d\,x^n)}\right]}{d^2\,e\,n} - \frac{2\,b\,x^{-3n}\,(e\,x)^{3n}\,\mathrm{PolyLog}\left[3, -e^{i\,(c+d\,x^n)}\right]}{d^3\,e\,n} + \frac{2\,b\,x^{-3n}\,(e\,x)^{3n}\,\mathrm{PolyLog}\left[3, e^{i\,(c+d\,x^n)}\right]}{d^3\,e\,n}$$

Result (type 8, 24 leaves):

$$\int (e\,x)^{-1+3n}\left(a + b\,\mathrm{Csc}\left[c + d\,x^n\right]\right) dx$$

Problem 77: Result more than twice size of optimal antiderivative.

$$\int (e\,x)^{-1+2n}\left(a + b\,\mathrm{Csc}\left[c + d\,x^n\right]\right)^2 dx$$

Optimal (type 4, 214 leaves, 11 steps):

$$\frac{a^2\,(e\,x)^{2n}}{2\,e\,n} - \frac{4\,a\,b\,x^{-n}\,(e\,x)^{2n}\,\mathrm{ArcTanh}\left[e^{i\,(c+d\,x^n)}\right]}{d\,e\,n} - \frac{b^2\,x^{-n}\,(e\,x)^{2n}\,\mathrm{Cot}\left[c + d\,x^n\right]}{d\,e\,n} + \frac{b^2\,x^{-2n}\,(e\,x)^{2n}\,\mathrm{Log}\left[\mathrm{Sin}\left[c + d\,x^n\right]\right]}{d^2\,e\,n} + \frac{2\,i\,a\,b\,x^{-2n}\,(e\,x)^{2n}\,\mathrm{PolyLog}\left[2, -e^{i\,(c+d\,x^n)}\right]}{d^2\,e\,n} - \frac{2\,i\,a\,b\,x^{-2n}\,(e\,x)^{2n}\,\mathrm{PolyLog}\left[2, e^{i\,(c+d\,x^n)}\right]}{d^2\,e\,n}$$

Result (type 4, 687 leaves):

$$\frac{b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Cot}[c] \left(a+b \operatorname{Csc}[c+d x^n]\right)^2 \operatorname{Sin}[c+d x^n]^2}{d n \left(b+a \operatorname{Sin}[c+d x^n]\right)^2}+$$

$$\frac{x^{1-n} (e x)^{-1+2 n} \operatorname{Csc}\left[\frac{c}{2}\right] \left(a+b \operatorname{Csc}[c+d x^n]\right)^2 \operatorname{Sec}\left[\frac{c}{2}\right] \left(-2 b^2 \operatorname{Cos}[c]+a^2 d x^n \operatorname{Sin}[c]\right) \operatorname{Sin}[c+d x^n]^2}{4 d n \left(b+a \operatorname{Sin}[c+d x^n]\right)^2}+$$

$$\left(b^2 x^{1-2 n} (e x)^{-1+2 n} \operatorname{Csc}[c] \left(a+b \operatorname{Csc}[c+d x^n]\right)^2 \left(-d x^n \operatorname{Cos}[c]+\operatorname{Log}\left[\operatorname{Cos}[d x^n] \operatorname{Sin}[c]+\operatorname{Cos}[c] \operatorname{Sin}[d x^n]\right] \operatorname{Sin}[c]\right) \operatorname{Sin}[c+d x^n]^2\right) \Big/$$

$$\left(d^2 n \left(\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2\right) \left(b+a \operatorname{Sin}[c+d x^n]\right)^2\right)+\frac{b^2 x^{1-n} (e x)^{-1+2 n} \operatorname{Csc}\left[\frac{c}{2}\right] \operatorname{Csc}\left[\frac{c}{2}+\frac{d x^n}{2}\right] \left(a+b \operatorname{Csc}[c+d x^n]\right)^2 \operatorname{Sin}\left[\frac{d x^n}{2}\right] \operatorname{Sin}[c+d x^n]^2}{2 d n \left(b+a \operatorname{Sin}[c+d x^n]\right)^2}+$$

$$\frac{b^2 x^{1-n} (e x)^{-1+2 n} \left(a+b \operatorname{Csc}[c+d x^n]\right)^2 \operatorname{Sec}\left[\frac{c}{2}\right] \operatorname{Sec}\left[\frac{c}{2}+\frac{d x^n}{2}\right] \operatorname{Sin}\left[\frac{d x^n}{2}\right] \operatorname{Sin}[c+d x^n]^2}{2 d n \left(b+a \operatorname{Sin}[c+d x^n]\right)^2}+\frac{1}{d^2 n \left(b+a \operatorname{Sin}[c+d x^n]\right)^2}$$

$$2 a b x^{1-2 n} (e x)^{-1+2 n} \left(a+b \operatorname{Csc}[c+d x^n]\right)^2 \operatorname{Sin}[c+d x^n]^2 \left(-\frac{2 \operatorname{ArcTan}[\operatorname{Tan}[c]] \operatorname{ArcTanh}\left[\frac{-\operatorname{Cos}[c]+\operatorname{Sin}[c] \operatorname{Tan}\left[\frac{d x^n}{2}\right]}{\sqrt{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}\right]}{\sqrt{\operatorname{Cos}[c]^2+\operatorname{Sin}[c]^2}}+\right.$$

$$\frac{1}{\sqrt{1+\operatorname{Tan}[c]^2}}\left(\left(d x^n+\operatorname{ArcTan}[\operatorname{Tan}[c]]\right) \left(\operatorname{Log}\left[1-e^{i \left(d x^n+\operatorname{ArcTan}[\operatorname{Tan}[c]]\right)}\right]-\operatorname{Log}\left[1+e^{i \left(d x^n+\operatorname{ArcTan}[\operatorname{Tan}[c]]\right)}\right]\right)+\right.$$

$$\left.\left. i \left(\operatorname{PolyLog}\left[2,-e^{i \left(d x^n+\operatorname{ArcTan}[\operatorname{Tan}[c]]\right)}\right]-\operatorname{PolyLog}\left[2,e^{i \left(d x^n+\operatorname{ArcTan}[\operatorname{Tan}[c]]\right)}\right]\right)\right) \operatorname{Sec}[c]\right)$$

Problem 78: Unable to integrate problem.

$$\int (e x)^{-1+3 n} \left(a+b \operatorname{Csc}[c+d x^n]\right)^2 dx$$

Optimal (type 4, 377 leaves, 16 steps):

$$\frac{a^2 (e x)^{3 n}}{3 e n}-\frac{i b^2 x^{-n} (e x)^{3 n}}{d e n}-\frac{4 a b x^{-n} (e x)^{3 n} \operatorname{ArcTanh}\left[e^{i (c+d x^n)}\right]}{d e n}-\frac{b^2 x^{-n} (e x)^{3 n} \operatorname{Cot}[c+d x^n]}{d e n}+$$

$$\frac{2 b^2 x^{-2 n} (e x)^{3 n} \operatorname{Log}\left[1-e^{2 i (c+d x^n)}\right]}{d^2 e n}+\frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,-e^{i (c+d x^n)}\right]}{d^2 e n}-\frac{4 i a b x^{-2 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,e^{i (c+d x^n)}\right]}{d^2 e n}-$$

$$\frac{i b^2 x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[2,e^{2 i (c+d x^n)}\right]}{d^3 e n}-\frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,-e^{i (c+d x^n)}\right]}{d^3 e n}+\frac{4 a b x^{-3 n} (e x)^{3 n} \operatorname{PolyLog}\left[3,e^{i (c+d x^n)}\right]}{d^3 e n}$$

Result (type 8, 26 leaves) :

$$\int (e\,x)^{-1+3\,n}\,\left(a+b\,\mathrm{Csc}\left[c+d\,x^n\right]\right)^2\,dx$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \frac{(e\,x)^{-1+2\,n}}{a+b\,\mathrm{Csc}[c+d\,x^n]}\,dx$$

Optimal (type 4, 338 leaves, 12 steps) :

$$\frac{(e\,x)^{2\,n}}{2\,a\,e\,n}+\frac{i\,b\,x^{-n}\,(e\,x)^{2\,n}\,\mathrm{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}-\frac{i\,b\,x^{-n}\,(e\,x)^{2\,n}\,\mathrm{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}+$$
$$\frac{b\,x^{-2\,n}\,(e\,x)^{2\,n}\,\mathrm{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}-\frac{b\,x^{-2\,n}\,(e\,x)^{2\,n}\,\mathrm{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}$$

Result (type 4, 1131 leaves) :

$$\frac{x\,(e\,x)^{-1+2\,n}\,\text{Csc}[c+d\,x^n]\,\left(b+a\,\text{Sin}[c+d\,x^n]\right)}{2\,a\,n\,\left(a+b\,\text{Csc}[c+d\,x^n]\right)}-\frac{1}{a\,d^2\,n\,\left(a+b\,\text{Csc}[c+d\,x^n]\right)}$$

$$b\,x^{1-2\,n}\,(e\,x)^{-1+2\,n}\,\text{Csc}[c+d\,x^n]\left(\frac{\pi\,\text{ArcTan}\left[\frac{a+b\,\text{Tan}\left[\frac{1}{2}\,(c+d\,x^n)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}}+\frac{1}{\sqrt{a^2-b^2}}\left(2\left(-c+\frac{\pi}{2}-d\,x^n\right)\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\right.\right.$$

$$2\left(-c+\text{ArcCos}\left[-\frac{b}{a}\right]\right)\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]-2\,\mathrm{i}\right.$$

$$\left.\left(\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{a^2-b^2}\,e^{-\frac{1}{2}\,\mathrm{i}\,\left(-c+\frac{\pi}{2}-d\,x^n\right)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Sin}[c+d\,x^n]}}\right]+$$

$$\left(\text{ArcCos}\left[-\frac{b}{a}\right]+2\,\mathrm{i}\left(\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{a^2-b^2}\,e^{\frac{1}{2}\,\mathrm{i}\,\left(-c+\frac{\pi}{2}-d\,x^n\right)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Sin}[c+d\,x^n]}}\right]-\left(\text{ArcCos}\left[-\frac{b}{a}\right]+2\,\mathrm{i}\,\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\text{Log}\left[1-\frac{\left(b-\mathrm{i}\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{b}{a}\right]+2\,\mathrm{i}\,\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\text{Log}\left[1-\frac{\left(b+\mathrm{i}\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]+$$

$$\mathrm{i}\left(\text{PolyLog}\left[2,\frac{\left(b-\mathrm{i}\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]-\right.$$

$$\left.\left.\left.\text{PolyLog}\left[2,\frac{\left(b+\mathrm{i}\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]\right)\right)\right)\Bigg/\left(b+a\,\text{Sin}[c+d\,x^n]\right)$$

Problem 81: Unable to integrate problem.

$$\int\frac{(e\,x)^{-1+3\,n}}{a+b\,\text{Csc}[c+d\,x^n]}\,dx$$

Optimal (type 4, 499 leaves, 14 steps):

$$\frac{(e\,x)^{3n}}{3\,a\,e\,n}+\frac{i\,b\,x^{-n}\,(e\,x)^{3n}\,\text{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}-\frac{i\,b\,x^{-n}\,(e\,x)^{3n}\,\text{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}+\frac{2\,b\,x^{-2n}\,(e\,x)^{3n}\,\text{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}-$$

$$\frac{2\,b\,x^{-2n}\,(e\,x)^{3n}\,\text{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}+\frac{2\,i\,b\,x^{-3n}\,(e\,x)^{3n}\,\text{PolyLog}\left[3,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^3\,e\,n}-\frac{2\,i\,b\,x^{-3n}\,(e\,x)^{3n}\,\text{PolyLog}\left[3,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^3\,e\,n}$$

Result (type 8, 26 leaves):

$$\int\frac{(e\,x)^{-1+3n}}{a+b\,\text{Csc}[c+d\,x^n]}\,dx$$

Problem 83: Result more than twice size of optimal antiderivative.

$$\int\frac{(e\,x)^{-1+2n}}{(a+b\,\text{Csc}[c+d\,x^n])^2}\,dx$$

Optimal (type 4, 778 leaves, 23 steps):

$$\frac{(e\,x)^{2n}}{2\,a^2\,e\,n}-\frac{i\,b^3\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d\,e\,n}+\frac{2\,i\,b\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d\,e\,n}+$$

$$\frac{i\,b^3\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d\,e\,n}-\frac{2\,i\,b\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1-\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d\,e\,n}+\frac{b^2\,x^{-2n}\,(e\,x)^{2n}\,\text{Log}[b+a\,\text{Sin}[c+d\,x^n]]}{a^2\,(a^2-b^2)\,d^2\,e\,n}-$$

$$\frac{b^3\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d^2\,e\,n}+\frac{2\,b\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^2\,e\,n}+$$

$$\frac{b^3\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d^2\,e\,n}-\frac{2\,b\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,\frac{i\,a\,e^{i\,(c+d\,x^n)}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^2\,e\,n}-\frac{b^2\,x^{-n}\,(e\,x)^{2n}\,\text{Cos}[c+d\,x^n]}{a\,(a^2-b^2)\,d\,e\,n\,(b+a\,\text{Sin}[c+d\,x^n])}$$

Result (type 4, 2850 leaves):

$$-\frac{b^2\,x^{1-n}\,(e\,x)^{-1+2n}\,\text{Csc}\left[\frac{c}{2}\right]\,\text{Csc}[c+d\,x^n]^2\,\text{Sec}\left[\frac{c}{2}\right]\,(b\,\text{Cos}[c]+a\,\text{Sin}[d\,x^n])\,(b+a\,\text{Sin}[c+d\,x^n])}{2\,a^2\,(-a+b)\,(a+b)\,d\,n\,(a+b\,\text{Csc}[c+d\,x^n])^2}-$$

$$\frac{b^2 x^{1-n} (e x)^{-1+2 n} \text{Cot}[c] \text{Csc}[c+d x^n]^2 \left(b+a \text{Sin}[c+d x^n]\right)^2}{a^2 \left(-a^2+b^2\right) d n \left(a+b \text{Csc}[c+d x^n]\right)^2}-$$

$$\frac{2 b^3 x^{1-2 n} (e x)^{-1+2 n} \text{ArcTan}\left[\frac{a \text{Cos}[c+d x^n]+i \left(b+a \text{Sin}[c+d x^n]\right)}{\sqrt{-a^2+b^2}}\right] \text{Cot}[c] \text{Csc}[c+d x^n]^2 \left(b+a \text{Sin}[c+d x^n]\right)^2}{a^2 \left(a^2-b^2\right) \sqrt{-a^2+b^2} d^2 n \left(a+b \text{Csc}[c+d x^n]\right)^2}-$$

$$\frac{1}{\left(a^2-b^2\right) d^2 n \left(a+b \text{Csc}[c+d x^n]\right)^2} 2 b x^{1-2 n} (e x)^{-1+2 n} \text{Csc}[c+d x^n]^2 \left(\frac{\pi \text{ArcTan}\left[\frac{a+b \text{Tan}\left[\frac{1}{2}\left(c+d x^n\right)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}}+\right.$$

$$\frac{1}{\sqrt{a^2-b^2}} \left(2 \left(-c+\frac{\pi}{2}-d x^n\right) \text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-2 \left(-c+\text{ArcCos}\left[-\frac{b}{a}\right]\right) \text{ArcTanh}\left[\frac{(a-b) \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\right.$$

$$\left(\text{ArcCos}\left[-\frac{b}{a}\right]-2 i \left(\text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b) \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{a^2-b^2} e^{-\frac{1}{2} i \left(-c+\frac{\pi}{2}-d x^n\right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \text{Sin}[c+d x^n]}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]+\right.$$

$$\left.2 i \left(\text{ArcTanh}\left[\frac{(a+b) \text{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b) \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{a^2-b^2} e^{\frac{1}{2} i \left(-c+\frac{\pi}{2}-d x^n\right)}}{\sqrt{2} \sqrt{a} \sqrt{b+a \text{Sin}[c+d x^n]}}\right]-$$

$$\left(\text{ArcCos}\left[-\frac{b}{a}\right]+2 i \text{ArcTanh}\left[\frac{(a-b) \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right) \text{Log}\left[1-\frac{\left(b-i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{b}{a}\right]+2 i \text{ArcTanh}\left[\frac{(a-b) \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right) \text{Log}\left[1-\frac{\left(b+i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}\right]+$$

$$i \left(\text{PolyLog}\left[2,\frac{\left(b-i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}\right]-\right.$$

$$\left.\left.\left.\text{PolyLog}\left[2,\frac{\left(b+i \sqrt{a^2-b^2}\right) \left(a+b-\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}{a \left(a+b+\sqrt{a^2-b^2} \text{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d x^n\right)\right]\right)}\right]\right)\right)\right) \left(b+a \text{Sin}[c+d x^n]\right)^2+$$

$$\frac{1}{a^2\left(a^2-b^2\right)d^2\,n\left(a+b\,\mathrm{Csc}[c+d\,x^n]\right)^2}\,b^3\,x^{1-2n}\,(e\,x)^{-1+2n}\,\mathrm{Csc}\left[c+d\,x^n\right]^2\left(\frac{\pi\,\mathrm{ArcTan}\left[\frac{a+b\,\mathrm{Tan}\left[\frac{1}{2}\left(c+d\,x^n\right)\right]}{\sqrt{-a^2+b^2}}\right]}{\sqrt{-a^2+b^2}}+\right.$$

$$\frac{1}{\sqrt{a^2-b^2}}\left(2\left(-c+\frac{\pi}{2}-d\,x^n\right)\mathrm{ArcTanh}\left[\frac{(a+b)\,\mathrm{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-2\left(-c+\mathrm{ArcCos}\left[-\frac{b}{a}\right]\right)\mathrm{ArcTanh}\left[\frac{(a-b)\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\right.$$

$$\left(\mathrm{ArcCos}\left[-\frac{b}{a}\right]-2\,i\left(\mathrm{ArcTanh}\left[\frac{(a+b)\,\mathrm{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\mathrm{ArcTanh}\left[\frac{(a-b)\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\mathrm{Log}\left[\frac{\sqrt{a^2-b^2}\;e^{-\frac{1}{2}i\left(-c+\frac{\pi}{2}-d\,x^n\right)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\mathrm{Sin}[c+d\,x^n]}}\right]+\left(\mathrm{ArcCos}\left[-\frac{b}{a}\right]+\right.$$

$$\left.2\,i\left(\mathrm{ArcTanh}\left[\frac{(a+b)\,\mathrm{Cot}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\mathrm{ArcTanh}\left[\frac{(a-b)\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)\mathrm{Log}\left[\frac{\sqrt{a^2-b^2}\;e^{\frac{1}{2}i\left(-c+\frac{\pi}{2}-d\,x^n\right)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\mathrm{Sin}[c+d\,x^n]}}\right]-$$

$$\left(\mathrm{ArcCos}\left[-\frac{b}{a}\right]+2\,i\,\mathrm{ArcTanh}\left[\frac{(a-b)\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\mathrm{Log}\left[1-\frac{\left(b-i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]+$$

$$\left(-\mathrm{ArcCos}\left[-\frac{b}{a}\right]+2\,i\,\mathrm{ArcTanh}\left[\frac{(a-b)\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\mathrm{Log}\left[1-\frac{\left(b+i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]+$$

$$i\left(\mathrm{PolyLog}\left[2,\frac{\left(b-i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]-\right.$$

$$\left.\left.\left.\mathrm{PolyLog}\left[2,\frac{\left(b+i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\mathrm{Tan}\left[\frac{1}{2}\left(-c+\frac{\pi}{2}-d\,x^n\right)\right]\right)}\right]\right)\right)\right)\left(b+a\,\mathrm{Sin}\left[c+d\,x^n\right]\right)^2+$$

$$\left(x^{1-n}\,(e\,x)^{-1+2n}\,\mathrm{Csc}\left[\frac{c}{2}\right]\mathrm{Csc}\left[c+d\,x^n\right]^2\mathrm{Sec}\left[\frac{c}{2}\right]\left(-2\,b^2\,\mathrm{Cos}[c]+a^2\,d\,x^n\,\mathrm{Sin}[c]-b^2\,d\,x^n\,\mathrm{Sin}[c]\right)\left(b+a\,\mathrm{Sin}\left[c+d\,x^n\right]\right)^2\right)\Big/$$

$$\left(4\,a^2\,(a-b)\,(a+b)\,d\,n\left(a+b\,\mathrm{Csc}\left[c+d\,x^n\right]\right)^2\right)+$$

$$\left(b^2 x^{1-2n} (e x)^{-1+2n} \operatorname{Csc}[c] \operatorname{Csc}[c+d x^n]^2 \left(-a d x^n \operatorname{Cos}[c] + a \operatorname{Log}[b + a \operatorname{Cos}[d x^n] \operatorname{Sin}[c] + a \operatorname{Cos}[c] \operatorname{Sin}[d x^n]] \operatorname{Sin}[c] + \frac{2 i a b \operatorname{ArcTan}\left[\frac{i a \operatorname{Cos}[c] - i (-b + a \operatorname{Sin}[c]) \operatorname{Tan}\left[\frac{d x^n}{2}\right]}{\sqrt{-b^2 + a^2 \operatorname{Cos}[c]^2 + a^2 \operatorname{Sin}[c]^2}}\right] \operatorname{Cos}[c]}{\sqrt{-b^2 + a^2 \operatorname{Cos}[c]^2 + a^2 \operatorname{Sin}[c]^2}}\right) (b + a \operatorname{Sin}[c+d x^n])^2\right) \Big/ \left(a (a^2 - b^2) d^2 n (a + b \operatorname{Csc}[c + d x^n])^2 (a^2 \operatorname{Cos}[c]^2 + a^2 \operatorname{Sin}[c]^2)\right)$$

Problem 84: Unable to integrate problem.

$$\int \frac{(e x)^{-1+3n}}{(a + b \operatorname{Csc}[c + d x^n])^2} \, dx$$

Optimal (type 4, 1417 leaves, 32 steps):

$$\frac{(e x)^{3n}}{3 a^2 e n} - \frac{i b^2 x^{-n} (e x)^{3n}}{a^2 (a^2 - b^2) d e n} + \frac{2 b^2 x^{-2n} (e x)^{3n} \operatorname{Log}\left[1 + \frac{a e^{i (c+d x^n)}}{i b - \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} + \frac{2 b^2 x^{-2n} (e x)^{3n} \operatorname{Log}\left[1 + \frac{a e^{i (c+d x^n)}}{i b + \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^2 e n} -$$

$$\frac{i b^3 x^{-n} (e x)^{3n} \operatorname{Log}\left[1 - \frac{i a e^{i (c+d x^n)}}{b - \sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} + \frac{2 i b x^{-n} (e x)^{3n} \operatorname{Log}\left[1 - \frac{i a e^{i (c+d x^n)}}{b - \sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} + \frac{i b^3 x^{-n} (e x)^{3n} \operatorname{Log}\left[1 - \frac{i a e^{i (c+d x^n)}}{b + \sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d e n} -$$

$$\frac{2 i b x^{-n} (e x)^{3n} \operatorname{Log}\left[1 - \frac{i a e^{i (c+d x^n)}}{b + \sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d e n} - \frac{2 i b^2 x^{-3n} (e x)^{3n} \operatorname{PolyLog}\left[2, -\frac{a e^{i (c+d x^n)}}{i b - \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} - \frac{2 i b^2 x^{-3n} (e x)^{3n} \operatorname{PolyLog}\left[2, -\frac{a e^{i (c+d x^n)}}{i b + \sqrt{a^2-b^2}}\right]}{a^2 (a^2 - b^2) d^3 e n} -$$

$$\frac{2 b^3 x^{-2n} (e x)^{3n} \operatorname{PolyLog}\left[2, \frac{i a e^{i (c+d x^n)}}{b - \sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} + \frac{4 b x^{-2n} (e x)^{3n} \operatorname{PolyLog}\left[2, \frac{i a e^{i (c+d x^n)}}{b - \sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} + \frac{2 b^3 x^{-2n} (e x)^{3n} \operatorname{PolyLog}\left[2, \frac{i a e^{i (c+d x^n)}}{b + \sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^2 e n} -$$

$$\frac{4 b x^{-2n} (e x)^{3n} \operatorname{PolyLog}\left[2, \frac{i a e^{i (c+d x^n)}}{b + \sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^2 e n} - \frac{2 i b^3 x^{-3n} (e x)^{3n} \operatorname{PolyLog}\left[3, \frac{i a e^{i (c+d x^n)}}{b - \sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} + \frac{4 i b x^{-3n} (e x)^{3n} \operatorname{PolyLog}\left[3, \frac{i a e^{i (c+d x^n)}}{b - \sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} +$$

$$\frac{2 i b^3 x^{-3n} (e x)^{3n} \operatorname{PolyLog}\left[3, \frac{i a e^{i (c+d x^n)}}{b + \sqrt{-a^2+b^2}}\right]}{a^2 (-a^2 + b^2)^{3/2} d^3 e n} - \frac{4 i b x^{-3n} (e x)^{3n} \operatorname{PolyLog}\left[3, \frac{i a e^{i (c+d x^n)}}{b + \sqrt{-a^2+b^2}}\right]}{a^2 \sqrt{-a^2 + b^2} d^3 e n} - \frac{b^2 x^{-n} (e x)^{3n} \operatorname{Cos}[c + d x^n]}{a (a^2 - b^2) d e n (b + a \operatorname{Sin}[c + d x^n])}$$

Result (type 8, 26 leaves):

$$\int \frac{(e\,x)^{-1+3\,n}}{\left(a+b\,\text{Csc}\left[c+d\,x^n\right]\right)^2}\,dx$$

Test results for the 59 problems in "4.6.1.2 (d csc)^n (a+b csc)^m.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]^5}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 55 leaves, 6 steps):

$$\frac{3\,\text{ArcTanh}[\text{Cos}[x]]}{2\,a}-\frac{4\,\text{Cot}[x]}{a}-\frac{4\,\text{Cot}[x]^3}{3\,a}+\frac{3\,\text{Cot}[x]\,\text{Csc}[x]}{2\,a}+\frac{\text{Cot}[x]\,\text{Csc}[x]^3}{a+a\,\text{Csc}[x]}$$

Result (type 3, 113 leaves):

$$\frac{1}{24\,a}\left(-20\,\text{Cot}\left[\frac{x}{2}\right]+3\,\text{Csc}\left[\frac{x}{2}\right]^2+36\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]-36\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]-3\,\text{Sec}\left[\frac{x}{2}\right]^2+8\,\text{Csc}[x]^3\,\text{Sin}\left[\frac{x}{2}\right]^4+\frac{48\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}-\frac{1}{2}\,\text{Csc}\left[\frac{x}{2}\right]^4\,\text{Sin}[x]+20\,\text{Tan}\left[\frac{x}{2}\right]\right)$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]^3}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 27 leaves, 4 steps):

$$\frac{\text{ArcTanh}[\text{Cos}[x]]}{a}-\frac{\text{Cot}[x]}{a}-\frac{\text{Cot}[x]}{a+a\,\text{Csc}[x]}$$

Result (type 3, 63 leaves):

$$\frac{-\text{Cot}\left[\frac{x}{2}\right]+2\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]-2\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]+\frac{4\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]}+\text{Tan}\left[\frac{x}{2}\right]}{2\,a}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]^2}{a + a\,\text{Csc}[x]} \, dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$-\frac{\text{ArcTanh}[\text{Cos}[x]]}{a} + \frac{\text{Cot}[x]}{a + a\,\text{Csc}[x]}$$

Result (type 3, 44 leaves):

$$\frac{-\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right] + \text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right] - \frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]}}{a}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Csc}[x]}{a + a\,\text{Csc}[x]} \, dx$$

Optimal (type 3, 12 leaves, 1 step):

$$-\frac{\text{Cot}[x]}{a + a\,\text{Csc}[x]}$$

Result (type 3, 26 leaves):

$$\frac{2\,\text{Sin}\left[\frac{x}{2}\right]}{a\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a\,\text{Csc}[x])^{3/2}} \, dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{a + a\,\text{Csc}[x]}}\right]}{a^{3/2}} + \frac{5\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[x]}{\sqrt{2}\,\sqrt{a + a\,\text{Csc}[x]}}\right]}{2\,\sqrt{2}\,a^{3/2}} + \frac{\text{Cot}[x]}{2\,(a + a\,\text{Csc}[x])^{3/2}}$$

Result (type 3, 165 leaves):

$$-\left(\left(\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)\left(2-2\,\text{Csc}[x]+4\,\text{ArcTan}\left[\frac{-2+\sqrt{1+\text{Csc}[x]}}{\sqrt{-1+\text{Csc}[x]}}\right]\sqrt{-1+\text{Csc}[x]}\,(1+\text{Csc}[x])-4\,\text{ArcTan}\left[\frac{2+\sqrt{1+\text{Csc}[x]}}{\sqrt{-1+\text{Csc}[x]}}\right]\sqrt{-1+\text{Csc}[x]}\,(1+\text{Csc}[x])+5\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\text{Csc}[x]}}\right]\sqrt{-1+\text{Csc}[x]}\,\text{Csc}[x]\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2\right)\right)\Big/\left(4\left(a\,(1+\text{Csc}[x])\right)^{3/2}\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)\right)\right)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int\sqrt{\text{Csc}[e+f\,x]}\,\sqrt{a+a\,\text{Csc}[e+f\,x]}\,dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$-\frac{2\sqrt{a}\,\text{ArcSinh}\left[\frac{\sqrt{a}\,\text{Cot}[e+f\,x]}{\sqrt{a+a\,\text{Csc}[e+f\,x]}}\right]}{f}$$

Result (type 3, 108 leaves):

$$\left(2\,\text{Cot}[e+f\,x]\sqrt{a\,(1+\text{Csc}[e+f\,x])}\left(\text{Log}[1+\text{Csc}[e+f\,x]]-\text{Log}\left[\sqrt{\text{Csc}[e+f\,x]}+\text{Csc}[e+f\,x]^{3/2}+\sqrt{\text{Cot}[e+f\,x]^2}\,\sqrt{1+\text{Csc}[e+f\,x]}\right]\right)\right)\Big/\left(f\sqrt{\text{Cot}[e+f\,x]^2}\,\sqrt{1+\text{Csc}[e+f\,x]}\right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int\sqrt{-\text{Csc}[e+f\,x]}\,\sqrt{a-a\,\text{Csc}[e+f\,x]}\,dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$-\frac{2\sqrt{a}\,\text{ArcSinh}\left[\frac{\sqrt{a}\,\text{Cot}[e+f\,x]}{\sqrt{a-a\,\text{Csc}[e+f\,x]}}\right]}{f}$$

Result (type 3, 116 leaves):

$$\left(2 \sqrt{-\text{Csc}[e+f x]} \sqrt{a-a \text{Csc}[e+f x]} \left(\text{ArcSinh}\left[\text{Tan}\left[\frac{1}{2}(e+f x)\right]\right]+\text{Log}\left[1+\sqrt{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2}\right]-\text{Log}\left[\text{Tan}\left[\frac{1}{2}(e+f x)\right]\right]\right) \text{Tan}\left[\frac{1}{2}(e+f x)\right]\right) \Big/ \left(f \sqrt{\text{Sec}\left[\frac{1}{2}(e+f x)\right]^2} \left(-1+\text{Tan}\left[\frac{1}{2}(e+f x)\right]\right)\right)$$

Problem 21: Result unnecessarily involves higher level functions.

$$\int \text{Csc}[c+d x]^{4/3} \sqrt{a+a \text{Csc}[c+d x]} \, dx$$

Optimal (type 4, 254 leaves, 4 steps):

$$-\frac{6 a \text{Cos}[c+d x] \text{Csc}[c+d x]^{4/3}}{5 d \sqrt{a+a \text{Csc}[c+d x]}}-\left(4 \times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^2 \text{Cot}[c+d x] \left(1-\text{Csc}[c+d x]^{1/3}\right) \sqrt{\frac{1+\text{Csc}[c+d x]^{1/3}+\text{Csc}[c+d x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d x]^{1/3}\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d x]^{1/3}}\right], -7-4\sqrt{3}\right]\right) \Big/ \left(5 d \sqrt{\frac{1-\text{Csc}[c+d x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d x]^{1/3}\right)^2}} \left(a-a \text{Csc}[c+d x]\right) \sqrt{a+a \text{Csc}[c+d x]}\right)$$

Result (type 5, 102 leaves):

$$-\left(\left(2 \sqrt{a \left(1+\text{Csc}[c+d x]\right)} \left(3 \text{Csc}[c+d x]^{1/3}+2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, 1-\text{Csc}[c+d x]\right]\right) \left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\right) \Big/ \left(5 d \left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)\right)\right)$$

Problem 22: Result unnecessarily involves higher level functions.

$$\int \text{Csc}[c+d x]^{1/3} \sqrt{a+a \text{Csc}[c+d x]} \, dx$$

Optimal (type 4, 213 leaves, 3 steps):

$$-\left(\left(2\times 3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\,\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.\right.$$
$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\,\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)\right)$$

Result (type 5, 46 leaves):

$$-\frac{2\,a\,\text{Cot}[c+d\,x]\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]}{d\sqrt{a\left(1+\text{Csc}[c+d\,x]\right)}}$$

Problem 23: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{a+a\,\text{Csc}[c+d\,x]}}{\text{Csc}[c+d\,x]^{2/3}}\,dx$$

Optimal (type 4, 254 leaves, 4 steps):

$$-\frac{3\,a\,\text{Cos}[c+d\,x]\,\text{Csc}[c+d\,x]^{1/3}}{2\,d\sqrt{a+a\,\text{Csc}[c+d\,x]}}-\left(3^{3/4}\sqrt{2+\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\,\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2\,d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\,\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)$$

Result (type 5, 110 leaves):

$$-\left(\left(\sqrt{a\left(1+\text{Csc}[c+d\,x]\right)}\,\left(3+\text{Csc}[c+d\,x]^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]\right)\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\Big/\right.$$
$$\left.\left(2\,d\,\text{Csc}[c+d\,x]^{2/3}\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\right)$$

Problem 24: Result unnecessarily involves higher level functions.

$$\int\text{Csc}[c+d\,x]^{5/3}\sqrt{a+a\,\text{Csc}[c+d\,x]}\,dx$$

Optimal (type 4, 514 leaves, 6 steps):

$$\frac{24\,a\,\text{Cot}[c+d\,x]}{7\,d\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}}-\frac{6\,a\,\text{Cos}[c+d\,x]\,\text{Csc}[c+d\,x]^{5/3}}{7\,d\,\sqrt{a+a\,\text{Csc}[c+d\,x]}}-$$

$$\left(12\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\,\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(7\,d\,\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\,\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)+$$

$$\left(8\sqrt{2}\;3^{3/4}\,a^2\,\text{Cot}[c+d\,x]\,\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7\,d\,\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\,\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)$$

Result (type 5, 102 leaves):

$$-\left(\left(2\sqrt{a\,\left(1+\text{Csc}[c+d\,x]\right)}\,\left(3\,\text{Csc}[c+d\,x]^{2/3}+4\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]\right)\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\right)\Big/\left(7\,d\,\left(\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right)\right)\right)$$

Problem 25: Result unnecessarily involves higher level functions.

$$\int \text{Csc}[c+d\,x]^{2/3}\sqrt{a+a\,\text{Csc}[c+d\,x]}\,dx$$

Optimal (type 4, 470 leaves, 5 steps):

$$\frac{6\,a\,\mathrm{Cot}[c+d\,x]}{d\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}-\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^2\,\mathrm{Cot}[c+d\,x]\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(d\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)+$$

$$\left(2\sqrt{2}\;3^{3/4}\,a^2\,\mathrm{Cot}[c+d\,x]\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\;\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(d\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)$$

Result (type 5, 85 leaves) :

$$-\left(\left(2\sqrt{a\left(1+\mathrm{Csc}[c+d\,x]\right)}\;\mathrm{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\mathrm{Csc}[c+d\,x]\right]\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\Big/\right.$$

$$\left.\left(d\left(\mathrm{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\mathrm{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\right)\right)$$

Problem 26: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}{\mathrm{Csc}[c+d\,x]^{1/3}}\,dx$$

Optimal (type 4, 508 leaves, 6 steps) :

$$-\frac{3\,a\,\mathrm{Cot}[c+d\,x]}{d\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}-\frac{3\,a\,\mathrm{Cos}[c+d\,x]\,\mathrm{Csc}[c+d\,x]^{2/3}}{d\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}+$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^2\,\mathrm{Cot}[c+d\,x]\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2\,d\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)-$$

$$\left(\sqrt{2}\;3^{3/4}\,a^2\,\mathrm{Cot}[c+d\,x]\left(1-\mathrm{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\mathrm{Csc}[c+d\,x]^{1/3}+\mathrm{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\;\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{1-\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(d\sqrt{\frac{1-\mathrm{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\mathrm{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\mathrm{Csc}[c+d\,x]\right)\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}\right)$$

Result (type 5, 66 leaves):

$$\frac{-3\,a\,\mathrm{Cos}[c+d\,x]\,\mathrm{Csc}[c+d\,x]^{2/3}+a\,\mathrm{Cot}[c+d\,x]\,\mathrm{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\mathrm{Csc}[c+d\,x]\right]}{d\sqrt{a\left(1+\mathrm{Csc}[c+d\,x]\right)}}$$

Problem 27: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{a+a\,\mathrm{Csc}[c+d\,x]}}{\mathrm{Csc}[c+d\,x]^{4/3}}\,dx$$

Optimal (type 4, 552 leaves, 7 steps):

$$-\frac{15\,a\,\text{Cot}[c+d\,x]}{8\,d\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}}-\frac{3\,a\,\text{Cos}[c+d\,x]}{4\,d\,\text{Csc}[c+d\,x]^{1/3}\sqrt{a+a\,\text{Csc}[c+d\,x]}}-$$

$$\frac{15\,a\,\text{Cos}[c+d\,x]\,\text{Csc}[c+d\,x]^{2/3}}{8\,d\sqrt{a+a\,\text{Csc}[c+d\,x]}}+\left(15\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^2\,\text{Cot}[c+d\,x]\,\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(16\,d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)-$$

$$\left(5\times 3^{3/4}\,a^2\,\text{Cot}[c+d\,x]\,\left(1-\text{Csc}[c+d\,x]^{1/3}\right)\sqrt{\frac{1+\text{Csc}[c+d\,x]^{1/3}+\text{Csc}[c+d\,x]^{2/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}{1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(4\sqrt{2}\,d\sqrt{\frac{1-\text{Csc}[c+d\,x]^{1/3}}{\left(1+\sqrt{3}-\text{Csc}[c+d\,x]^{1/3}\right)^2}}\left(a-a\,\text{Csc}[c+d\,x]\right)\sqrt{a+a\,\text{Csc}[c+d\,x]}\right)$$

Result (type 5, 118 leaves):

$$\frac{1}{8\,d\sqrt{a\left(1+\text{Csc}[c+d\,x]\right)}}a\,\text{Csc}[c+d\,x]^{2/3}\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]-\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)$$

$$\left(\text{Cos}\left[\frac{1}{2}(c+d\,x)\right]+\text{Sin}\left[\frac{1}{2}(c+d\,x)\right]\right)\left(-15+5\,\text{Csc}[c+d\,x]^{1/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\text{Csc}[c+d\,x]\right]-6\,\text{Sin}[c+d\,x]\right)$$

Problem 33: Unable to integrate problem.

$$\int (a+a\,\text{Csc}[e+f\,x])^m\,dx$$

Optimal (type 6, 84 leaves, 3 steps):

$$-\left(\left(\sqrt{2}\,\text{AppellF1}\left[\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}\left(1+\text{Csc}[e+f\,x]\right),1+\text{Csc}[e+f\,x]\right]\text{Cot}[e+f\,x]\left(a+a\,\text{Csc}[e+f\,x]\right)^m\right)\Big/\left(f\,(1+2\,m)\sqrt{1-\text{Csc}[e+f\,x]}\right)\right)$$

Result (type 8, 14 leaves):

$$\int (a+a\,\text{Csc}[e+f\,x])^m\,dx$$

Problem 34: Unable to integrate problem.

$$\int (a + a \operatorname{Csc}[e + f x])^m \operatorname{Sin}[e + f x] \, dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 2, \frac{3}{2} + m, \frac{1}{2} (1 + \operatorname{Csc}[e + f x]), 1 + \operatorname{Csc}[e + f x]\right] \operatorname{Cot}[e + f x] (a + a \operatorname{Csc}[e + f x])^m}{f (1 + 2 m) \sqrt{1 - \operatorname{Csc}[e + f x]}}$$

Result (type 8, 21 leaves):

$$\int (a + a \operatorname{Csc}[e + f x])^m \operatorname{Sin}[e + f x] \, dx$$

Problem 35: Unable to integrate problem.

$$\int (a + a \operatorname{Csc}[e + f x])^m \operatorname{Sin}[e + f x]^2 \, dx$$

Optimal (type 6, 84 leaves, 3 steps):

$$-\left(\left(\sqrt{2} \operatorname{AppellF1}\left[\frac{1}{2} + m, \frac{1}{2}, 3, \frac{3}{2} + m, \frac{1}{2} (1 + \operatorname{Csc}[e + f x]), 1 + \operatorname{Csc}[e + f x]\right] \operatorname{Cot}[e + f x] (a + a \operatorname{Csc}[e + f x])^m\right) \Big/ \left(f (1 + 2 m) \sqrt{1 - \operatorname{Csc}[e + f x]}\right)\right)$$

Result (type 8, 23 leaves):

$$\int (a + a \operatorname{Csc}[e + f x])^m \operatorname{Sin}[e + f x]^2 \, dx$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{Csc}[c + d x])^4 \, dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$a^4 x - \frac{2 a b (2 a^2 + b^2) \operatorname{ArcTanh}[\operatorname{Cos}[c + d x]]}{d} - \frac{b^2 (17 a^2 + 2 b^2) \operatorname{Cot}[c + d x]}{3 d} - \frac{4 a b^3 \operatorname{Cot}[c + d x] \operatorname{Csc}[c + d x]}{3 d} - \frac{b^2 \operatorname{Cot}[c + d x] (a + b \operatorname{Csc}[c + d x])^2}{3 d}$$

Result (type 3, 568 leaves):

$$\frac{a^4 (c+d x) (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4}{d (b+a \operatorname{Sin}[c+d x])^4}+\frac{\left(-9 a^2 b^2 \operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]-b^4 \operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right) \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right] (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4}{3 d (b+a \operatorname{Sin}[c+d x])^4}-$$

$$\frac{a b^3 \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4}{2 d (b+a \operatorname{Sin}[c+d x])^4}-\frac{b^4 \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right] \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sin}[c+d x]^4}{24 d (b+a \operatorname{Sin}[c+d x])^4}-$$

$$\frac{2 \left(2 a^3 b+a b^3\right) (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right] \operatorname{Sin}[c+d x]^4}{d (b+a \operatorname{Sin}[c+d x])^4}+$$

$$\frac{2 \left(2 a^3 b+a b^3\right) (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right] \operatorname{Sin}[c+d x]^4}{d (b+a \operatorname{Sin}[c+d x])^4}+\frac{a b^3 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Sin}[c+d x]^4}{2 d (b+a \operatorname{Sin}[c+d x])^4}+$$

$$\frac{(a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right] \left(9 a^2 b^2 \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]+b^4 \operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right) \operatorname{Sin}[c+d x]^4}{3 d (b+a \operatorname{Sin}[c+d x])^4}+$$

$$\frac{b^4 (a+b \operatorname{Csc}[c+d x])^4 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2 \operatorname{Sin}[c+d x]^4 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{24 d (b+a \operatorname{Sin}[c+d x])^4}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int (a+b \operatorname{Csc}[c+d x])^3 \, dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$a^3 x-\frac{b \left(6 a^2+b^2\right) \operatorname{ArcTanh}[\operatorname{Cos}[c+d x]]}{2 d}-\frac{5 a b^2 \operatorname{Cot}[c+d x]}{2 d}-\frac{b^2 \operatorname{Cot}[c+d x] (a+b \operatorname{Csc}[c+d x])}{2 d}$$

Result (type 3, 152 leaves):

$$\frac{1}{8 d}\left(8 a^3 c+8 a^3 d x-12 a b^2 \operatorname{Cot}\left[\frac{1}{2} (c+d x)\right]-b^3 \operatorname{Csc}\left[\frac{1}{2} (c+d x)\right]^2-24 a^2 b \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right]-4 b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} (c+d x)\right]\right]+\right.$$
$$\left.24 a^2 b \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right]+4 b^3 \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2} (c+d x)\right]\right]+b^3 \operatorname{Sec}\left[\frac{1}{2} (c+d x)\right]^2+12 a b^2 \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int (a+b \operatorname{Csc}[c+d x])^2 \, dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$a^2 x - \frac{2 a b \text{ArcTanh}[\text{Cos}[c + d x]]}{d} - \frac{b^2 \text{Cot}[c + d x]}{d}$$

Result (type 3, 76 leaves):

$$\frac{1}{2 d}\left(-b^2 \text{Cot}\left[\frac{1}{2}(c + d x)\right] + 2 a \left(a c + a d x - 2 b \text{Log}\left[\text{Cos}\left[\frac{1}{2}(c + d x)\right]\right] + 2 b \text{Log}\left[\text{Sin}\left[\frac{1}{2}(c + d x)\right]\right]\right) + b^2 \text{Tan}\left[\frac{1}{2}(c + d x)\right]\right)$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{3 + 5 \text{Csc}[c + d x]} \, dx$$

Optimal (type 3, 31 leaves, 2 steps):

$$-\frac{x}{12} - \frac{5 \text{ArcTan}\left[\frac{\text{Cos}[c+d x]}{3+\text{Sin}[c+d x]}\right]}{6 d}$$

Result (type 3, 66 leaves):

$$\frac{2 (c + d x) - 5 \text{ArcTan}\left[\frac{2 \left(\text{Cos}\left[\frac{1}{2}(c+d x)\right]+\text{Sin}\left[\frac{1}{2}(c+d x)\right]\right)}{\text{Cos}\left[\frac{1}{2}(c+d x)\right]-\text{Sin}\left[\frac{1}{2}(c+d x)\right]}\right]}{6 d}$$

Problem 54: Unable to integrate problem.

$$\int \text{Csc}[e + f x]^3 (a + b \text{Csc}[e + f x])^m \, dx$$

Optimal (type 6, 274 leaves, 8 steps):

$$-\frac{\text{Cot}[e + f x] (a + b \text{Csc}[e + f x])^{1+m}}{b f (2 + m)} + \left(\sqrt{2}\, a (a + b) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1 - m, \frac{3}{2}, \frac{1}{2}(1 - \text{Csc}[e + f x]), \frac{b (1 - \text{Csc}[e + f x])}{a + b}\right] \text{Cot}[e + f x] (a + b \text{Csc}[e + f x])^m \left(\frac{a + b \text{Csc}[e + f x]}{a + b}\right)^{-m}\right) \Big/ \left(b^2 f (2 + m) \sqrt{1 + \text{Csc}[e + f x]}\right) - \left(\sqrt{2} \left(a^2 + b^2 (1 + m)\right) \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1 - \text{Csc}[e + f x]), \frac{b (1 - \text{Csc}[e + f x])}{a + b}\right] \text{Cot}[e + f x] (a + b \text{Csc}[e + f x])^m \left(\frac{a + b \text{Csc}[e + f x]}{a + b}\right)^{-m}\right) \Big/ \left(b^2 f (2 + m) \sqrt{1 + \text{Csc}[e + f x]}\right)$$

Result (type 8, 23 leaves):

$$\int \text{Csc}[e+f x]^3 \, (a+b\,\text{Csc}[e+f x])^m \, dx$$

Problem 55: Unable to integrate problem.

$$\int \text{Csc}[e+f x]^2 \, (a+b\,\text{Csc}[e+f x])^m \, dx$$

Optimal (type 6, 220 leaves, 7 steps):

$$-\frac{1}{b f \sqrt{1+\text{Csc}[e+f x]}} \sqrt{2} \, (a+b) \, \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -1-m, \frac{3}{2}, \frac{1}{2} (1-\text{Csc}[e+f x]), \frac{b (1-\text{Csc}[e+f x])}{a+b}\right] \text{Cot}[e+f x] \, (a+b\,\text{Csc}[e+f x])^m \left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m} + \frac{1}{b f \sqrt{1+\text{Csc}[e+f x]}} \sqrt{2} \, a \, \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1-\text{Csc}[e+f x]), \frac{b (1-\text{Csc}[e+f x])}{a+b}\right] \text{Cot}[e+f x] \, (a+b\,\text{Csc}[e+f x])^m \left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m}$$

Result (type 8, 23 leaves):

$$\int \text{Csc}[e+f x]^2 \, (a+b\,\text{Csc}[e+f x])^m \, dx$$

Problem 56: Unable to integrate problem.

$$\int \text{Csc}[e+f x] \, (a+b\,\text{Csc}[e+f x])^m \, dx$$

Optimal (type 6, 104 leaves, 3 steps):

$$-\frac{1}{f \sqrt{1+\text{Csc}[e+f x]}} \sqrt{2} \, \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1-\text{Csc}[e+f x]), \frac{b (1-\text{Csc}[e+f x])}{a+b}\right] \text{Cot}[e+f x] \, (a+b\,\text{Csc}[e+f x])^m \left(\frac{a+b\,\text{Csc}[e+f x]}{a+b}\right)^{-m}$$

Result (type 8, 21 leaves):

$$\int \text{Csc}[e+f x] \, (a+b\,\text{Csc}[e+f x])^m \, dx$$

Test results for the 16 problems in "4.6.1.3 (d cos)^n (a+b csc)^m.m"

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]^2}{a + a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 23 leaves, 6 steps):

$$\frac{\text{Sec}[x]^3}{3\,a} - \frac{\text{Tan}[x]^3}{3\,a}$$

Result (type 3, 56 leaves):

$$-\frac{-3 + \text{Cos}[2\,x] - 2\,\text{Sin}[x] + \text{Cos}[x]\,\left(1 + \text{Sin}[x]\right)}{6\,a\,\left(\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right)\,\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^3}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}[x]^4}{a + a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 34 leaves, 7 steps):

$$\frac{\text{Sec}[x]^5}{5\,a} - \frac{\text{Tan}[x]^3}{3\,a} - \frac{\text{Tan}[x]^5}{5\,a}$$

Result (type 3, 85 leaves):

$$-\left(\left(-240 + 54\,\text{Cos}[x] + 32\,\text{Cos}[2\,x] + 18\,\text{Cos}[3\,x] + 16\,\text{Cos}[4\,x] - 96\,\text{Sin}[x] + 18\,\text{Sin}[2\,x] - 32\,\text{Sin}[3\,x] + 9\,\text{Sin}[4\,x]\right) \Big/ \left(960\,a\,\left(\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right)^3\,\left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right)^5\right)\right)$$

Test results for the 23 problems in "4.6.1.4 (d cot)^n (a+b csc)^m.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tan}[x]^4}{a + a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$\frac{x}{a}-\frac{(15-8\,\text{Csc}[x])\,\text{Tan}[x]}{15\,a}+\frac{(5-4\,\text{Csc}[x])\,\text{Tan}[x]^3}{15\,a}-\frac{(1-\text{Csc}[x])\,\text{Tan}[x]^5}{5\,a}$$

Result (type 3, 111 leaves):

$$\left(200+6\,(-89+120\,x)\,\text{Cos}[x]+128\,\text{Cos}[2\,x]-178\,\text{Cos}[3\,x]+240\,x\,\text{Cos}[3\,x]+184\,\text{Cos}[4\,x]-64\,\text{Sin}[x]-178\,\text{Sin}[2\,x]+240\,x\,\text{Sin}[2\,x]-128\,\text{Sin}[3\,x]-89\,\text{Sin}[4\,x]+120\,x\,\text{Sin}[4\,x]\right)\Big/\left(960\,a\,\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^3\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^5\right)$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cot}[x]}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\frac{\text{Log}[1+\text{Sin}[x]]}{a}$$

Result (type 3, 19 leaves):

$$\frac{2\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right]}{a}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cot}[x]^3}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$-\frac{\text{Csc}[x]}{a}-\frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 35 leaves):

$$-\frac{\text{Cot}\left[\frac{x}{2}\right]}{2\,a}-\frac{\text{Log}[\text{Sin}[x]]}{a}-\frac{\text{Tan}\left[\frac{x}{2}\right]}{2\,a}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cot}[x]^4}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 31 leaves, 4 steps):

$$\frac{x}{a}+\frac{\text{ArcTanh}[\text{Cos}[x]]}{2a}+\frac{\text{Cot}[x]\,(2-\text{Csc}[x])}{2a}$$

Result (type 3, 90 leaves):

$$\frac{x}{a}+\frac{\text{Cot}\left[\frac{x}{2}\right]}{2a}-\frac{\text{Csc}\left[\frac{x}{2}\right]^2}{8a}+\frac{\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]}{2a}-\frac{\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]}{2a}+\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{8a}-\frac{\text{Tan}\left[\frac{x}{2}\right]}{2a}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cot}[x]^5}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 36 leaves, 3 steps):

$$\frac{\text{Csc}[x]}{a}+\frac{\text{Csc}[x]^2}{2a}-\frac{\text{Csc}[x]^3}{3a}+\frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 106 leaves):

$$\frac{5\,\text{Cot}\left[\frac{x}{2}\right]}{12a}+\frac{\text{Csc}\left[\frac{x}{2}\right]^2}{8a}-\frac{\text{Cot}\left[\frac{x}{2}\right]\text{Csc}\left[\frac{x}{2}\right]^2}{24a}+\frac{\text{Log}[\text{Sin}[x]]}{a}+\frac{\text{Sec}\left[\frac{x}{2}\right]^2}{8a}+\frac{5\,\text{Tan}\left[\frac{x}{2}\right]}{12a}-\frac{\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{24a}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cot}[x]^6}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{x}{a}-\frac{3\,\text{ArcTanh}[\text{Cos}[x]]}{8a}+\frac{\text{Cot}[x]^3\,(4-3\,\text{Csc}[x])}{12a}-\frac{\text{Cot}[x]\,(8-3\,\text{Csc}[x])}{8a}$$

Result (type 3, 163 leaves):

$$-\frac{x}{a}-\frac{2\,\text{Cot}\left[\frac{x}{2}\right]}{3a}+\frac{5\,\text{Csc}\left[\frac{x}{2}\right]^2}{32a}+\frac{\text{Cot}\left[\frac{x}{2}\right]\text{Csc}\left[\frac{x}{2}\right]^2}{24a}-\frac{\text{Csc}\left[\frac{x}{2}\right]^4}{64a}-\frac{3\,\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right]\right]}{8a}+\frac{3\,\text{Log}\left[\text{Sin}\left[\frac{x}{2}\right]\right]}{8a}-\frac{5\,\text{Sec}\left[\frac{x}{2}\right]^2}{32a}+\frac{\text{Sec}\left[\frac{x}{2}\right]^4}{64a}+\frac{2\,\text{Tan}\left[\frac{x}{2}\right]}{3a}-\frac{\text{Sec}\left[\frac{x}{2}\right]^2\text{Tan}\left[\frac{x}{2}\right]}{24a}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Cot}[x]^7}{a+a\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 58 leaves, 3 steps):

$$-\frac{\text{Csc}[x]}{a}-\frac{\text{Csc}[x]^2}{a}+\frac{2\,\text{Csc}[x]^3}{3\,a}+\frac{\text{Csc}[x]^4}{4\,a}-\frac{\text{Csc}[x]^5}{5\,a}-\frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 179 leaves):

$$-\frac{89\,\text{Cot}\left[\frac{x}{2}\right]}{240\,a}-\frac{7\,\text{Csc}\left[\frac{x}{2}\right]^2}{32\,a}+\frac{31\,\text{Cot}\left[\frac{x}{2}\right]\,\text{Csc}\left[\frac{x}{2}\right]^2}{480\,a}+\frac{\text{Csc}\left[\frac{x}{2}\right]^4}{64\,a}-\frac{\text{Cot}\left[\frac{x}{2}\right]\,\text{Csc}\left[\frac{x}{2}\right]^4}{160\,a}-\frac{\text{Log}[\text{Sin}[x]]}{a}-\frac{7\,\text{Sec}\left[\frac{x}{2}\right]^2}{32\,a}+\frac{\text{Sec}\left[\frac{x}{2}\right]^4}{64\,a}-\frac{89\,\text{Tan}\left[\frac{x}{2}\right]}{240\,a}+\frac{31\,\text{Sec}\left[\frac{x}{2}\right]^2\,\text{Tan}\left[\frac{x}{2}\right]}{480\,a}-\frac{\text{Sec}\left[\frac{x}{2}\right]^4\,\text{Tan}\left[\frac{x}{2}\right]}{160\,a}$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Tan}[x]^5}{a+b\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 178 leaves, 3 steps):

$$\frac{1}{16\,(a+b)\,(1-\text{Csc}[x])^2}+\frac{5\,a+7\,b}{16\,(a+b)^2\,(1-\text{Csc}[x])}+\frac{1}{16\,(a-b)\,(1+\text{Csc}[x])^2}+\frac{5\,a-7\,b}{16\,(a-b)^2\,(1+\text{Csc}[x])}-\frac{\left(8\,a^2+21\,a\,b+15\,b^2\right)\,\text{Log}[1-\text{Csc}[x]]}{16\,(a+b)^3}-\frac{\left(8\,a^2-21\,a\,b+15\,b^2\right)\,\text{Log}[1+\text{Csc}[x]]}{16\,(a-b)^3}+\frac{b^6\,\text{Log}[a+b\,\text{Csc}[x]]}{a\,\left(a^2-b^2\right)^3}-\frac{\text{Log}[\text{Sin}[x]]}{a}$$

Result (type 3, 301 leaves):

$$\frac{1}{16\,(a+b\,\text{Csc}[x])}\text{Csc}[x]\left(-\frac{32\,i\,\left(a^5-3\,a^3\,b^2+3\,a\,b^4\right)\,x}{(a-b)^3\,(a+b)^3}-\frac{2\,i\,\left(8\,a^2-21\,a\,b+15\,b^2\right)\,\text{ArcTan}[\text{Cot}[x]]}{(a-b)^3}-\frac{2\,i\,\left(8\,a^2+21\,a\,b+15\,b^2\right)\,\text{ArcTan}[\text{Cot}[x]]}{(a+b)^3}+\frac{\left(8\,a^2-21\,a\,b+15\,b^2\right)\,\text{Log}\left[\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2\right]}{(-a+b)^3}-\frac{\left(8\,a^2+21\,a\,b+15\,b^2\right)\,\text{Log}[1-\text{Sin}[x]]}{(a+b)^3}+\frac{16\,b^6\,\text{Log}[b+a\,\text{Sin}[x]]}{a\,\left(a^2-b^2\right)^3}+\frac{1}{(a+b)\,\left(\text{Cos}\left[\frac{x}{2}\right]-\text{Sin}\left[\frac{x}{2}\right]\right)^4}+\frac{1}{(a-b)\,\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^4}+\frac{-7\,a+9\,b}{(a-b)^2\,\left(\text{Cos}\left[\frac{x}{2}\right]+\text{Sin}\left[\frac{x}{2}\right]\right)^2}+\frac{7\,a+9\,b}{(a+b)^2\,(-1+\text{Sin}[x])}\right)(b+a\,\text{Sin}[x])$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]^5}{a+b\,\text{Csc}[x]}\,dx$$

Optimal (type 3, 72 leaves, 3 steps):

$$-\frac{(a^2-2b^2)\,Csc[x]}{b^3}+\frac{a\,Csc[x]^2}{2b^2}-\frac{Csc[x]^3}{3b}+\frac{(a^2-b^2)^2\,Log[a+b\,Csc[x]]}{a\,b^4}+\frac{Log[Sin[x]]}{a}$$

Result (type 3, 179 leaves):

$$\frac{1}{48\,a\,b^4}\left((-24a^3b+44ab^3)\,Cot\left[\frac{x}{2}\right]+6a^2b^2\,Csc\left[\frac{x}{2}\right]^2-48a^4\,Log[Sin[x]]+96a^2b^2\,Log[Sin[x]]+48a^4\,Log[b+a\,Sin[x]]-96a^2b^2\,Log[b+a\,Sin[x]]+48b^4\,Log[b+a\,Sin[x]]+6a^2b^2\,Sec\left[\frac{x}{2}\right]^2-16ab^3\,Csc[x]^3\,Sin\left[\frac{x}{2}\right]^4-a\,b^3\,Csc\left[\frac{x}{2}\right]^4\,Sin[x]-24a^3b\,Tan\left[\frac{x}{2}\right]+44ab^3\,Tan\left[\frac{x}{2}\right]\right)$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int\frac{Cot[x]^7}{a+b\,Csc[x]}\,dx$$

Optimal (type 3, 122 leaves, 3 steps):

$$-\frac{(a^4-3a^2b^2+3b^4)\,Csc[x]}{b^5}+\frac{a\,(a^2-3b^2)\,Csc[x]^2}{2b^4}-\frac{(a^2-3b^2)\,Csc[x]^3}{3b^3}+\frac{a\,Csc[x]^4}{4b^2}-\frac{Csc[x]^5}{5b}+\frac{(a^2-b^2)^3\,Log[a+b\,Csc[x]]}{a\,b^6}-\frac{Log[Sin[x]]}{a}$$

Result (type 3, 343 leaves):

$$-\frac{1}{960\,a\,b^6}\left(4ab\,(120a^4-340a^2b^2+309b^4)\,Cot\left[\frac{x}{2}\right]-30a^2b^2\,(4a^2-11b^2)\,Csc\left[\frac{x}{2}\right]^2+960a^6\,Log[Sin[x]]-2880a^4b^2\,Log[Sin[x]]+2880a^2b^4\,Log[Sin[x]]-960a^6\,Log[b+a\,Sin[x]]+2880a^4b^2\,Log[b+a\,Sin[x]]-2880a^2b^4\,Log[b+a\,Sin[x]]+960b^6\,Log[b+a\,Sin[x]]-120a^4b^2\,Sec\left[\frac{x}{2}\right]^2+330a^2b^4\,Sec\left[\frac{x}{2}\right]^2-15a^2b^4\,Sec\left[\frac{x}{2}\right]^4+320a^3b^3\,Csc[x]^3\,Sin\left[\frac{x}{2}\right]^4-816ab^5\,Csc[x]^3\,Sin\left[\frac{x}{2}\right]^4+3ab^5\,Csc\left[\frac{x}{2}\right]^6\,Sin[x]+a\,b^3\,Csc\left[\frac{x}{2}\right]^4\,(-15ab+20a^2\,Sin[x]-51b^2\,Sin[x])+480a^5b\,Tan\left[\frac{x}{2}\right]-1360a^3b^3\,Tan\left[\frac{x}{2}\right]+1236ab^5\,Tan\left[\frac{x}{2}\right]+6ab^5\,Sec\left[\frac{x}{2}\right]^4\,Tan\left[\frac{x}{2}\right]\right)$$

Test results for the 24 problems in "4.6.3.1 (a+b csc)^m (d csc)^n (A+B csc).m"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int Csc[c+d\,x]\,(a+a\,Csc[c+d\,x])\,(A+A\,Csc[c+d\,x])\,dx$$

Optimal (type 3, 51 leaves, 6 steps):

$$-\frac{3\,a\,A\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{2\,d}-\frac{2\,a\,A\,\text{Cot}[c+d\,x]}{d}-\frac{a\,A\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]}{2\,d}$$

Result (type 3, 137 leaves):

$$-\frac{2\,a\,A\,\text{Cot}[c+d\,x]}{d}-\frac{a\,A\,\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}-\frac{a\,A\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}-\frac{a\,A\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}+\frac{a\,A\,\text{Log}\left[\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{a\,A\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}+\frac{a\,A\,\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int (a+a\,\text{Csc}[c+d\,x])\,(A+A\,\text{Csc}[c+d\,x])\,\text{Sin}[c+d\,x]\,dx$$

Optimal (type 3, 33 leaves, 5 steps):

$$2\,a\,A\,x-\frac{a\,A\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}-\frac{a\,A\,\text{Cos}[c+d\,x]}{d}$$

Result (type 3, 72 leaves):

$$2\,a\,A\,x-\frac{a\,A\,\text{Cos}[c]\,\text{Cos}[d\,x]}{d}-\frac{a\,A\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{a\,A\,\text{Log}\left[\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{a\,A\,\text{Sin}[c]\,\text{Sin}[d\,x]}{d}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]\,(a-a\,\text{Csc}[c+d\,x])\,(A+A\,\text{Csc}[c+d\,x])\,dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$-\frac{a\,A\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{2\,d}+\frac{a\,A\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]}{2\,d}$$

Result (type 3, 79 leaves):

$$-a\,A\left(-\frac{\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}+\frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}-\frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}+\frac{\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}\right)$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]\,(a+a\,\text{Csc}[c+d\,x])\,(A-A\,\text{Csc}[c+d\,x])\,dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$-\frac{a\,A\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{2\,d}+\frac{a\,A\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]}{2\,d}$$

Result (type 3, 79 leaves):

$$-a\,A\left(-\frac{\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}+\frac{\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}-\frac{\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}+\frac{\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}\right)$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \text{Csc}[c+d\,x]\,(a-a\,\text{Csc}[c+d\,x])\,(A-A\,\text{Csc}[c+d\,x])\,dx$$

Optimal (type 3, 51 leaves, 6 steps):

$$-\frac{3\,a\,A\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{2\,d}+\frac{2\,a\,A\,\text{Cot}[c+d\,x]}{d}-\frac{a\,A\,\text{Cot}[c+d\,x]\,\text{Csc}[c+d\,x]}{2\,d}$$

Result (type 3, 137 leaves):

$$\frac{2\,a\,A\,\text{Cot}[c+d\,x]}{d}-\frac{a\,A\,\text{Csc}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}-\frac{a\,A\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}-\frac{a\,A\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}+\frac{a\,A\,\text{Log}\left[\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{a\,A\,\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,(c+d\,x)\right]\right]}{2\,d}+\frac{a\,A\,\text{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^2}{8\,d}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int (a-a\,\text{Csc}[c+d\,x])\,(A-A\,\text{Csc}[c+d\,x])\,\text{Sin}[c+d\,x]\,dx$$

Optimal (type 3, 33 leaves, 5 steps):

$$-2\,a\,A\,x-\frac{a\,A\,\text{ArcTanh}[\text{Cos}[c+d\,x]]}{d}-\frac{a\,A\,\text{Cos}[c+d\,x]}{d}$$

Result (type 3, 72 leaves):

$$-2\,a\,A\,x-\frac{a\,A\,\text{Cos}[c]\,\text{Cos}[d\,x]}{d}-\frac{a\,A\,\text{Log}\left[\text{Cos}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{a\,A\,\text{Log}\left[\text{Sin}\left[\frac{c}{2}+\frac{d\,x}{2}\right]\right]}{d}+\frac{a\,A\,\text{Sin}[c]\,\text{Sin}[d\,x]}{d}$$

Test results for the 1 problems in "4.6.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2).m"

Test results for the 27 problems in "4.6.7 (d trig)^m (a+b (c csc)^n)^p.m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\,\text{Csc}[c+d\,x]^2\right)^3\,dx$$

Optimal (type 3, 74 leaves, 4 steps):

$$a^3\,x-\frac{b\,\left(3\,a^2+3\,a\,b+b^2\right)\,\text{Cot}[c+d\,x]}{d}-\frac{b^2\,\left(3\,a+2\,b\right)\,\text{Cot}[c+d\,x]^3}{3\,d}-\frac{b^3\,\text{Cot}[c+d\,x]^5}{5\,d}$$

Result (type 3, 266 leaves):

$$\frac{8\,b^3\,\text{Cos}[c+d\,x]\,\left(a+b\,\text{Csc}[c+d\,x]^2\right)^3\,\text{Sin}[c+d\,x]}{5\,d\,\left(-a-2\,b+a\,\text{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^3}+\frac{8\,\left(15\,a\,b^2\,\text{Cos}[c+d\,x]+4\,b^3\,\text{Cos}[c+d\,x]\right)\,\left(a+b\,\text{Csc}[c+d\,x]^2\right)^3\,\text{Sin}[c+d\,x]^3}{15\,d\,\left(-a-2\,b+a\,\text{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^3}+\frac{8\,\left(45\,a^2\,b\,\text{Cos}[c+d\,x]+30\,a\,b^2\,\text{Cos}[c+d\,x]+8\,b^3\,\text{Cos}[c+d\,x]\right)\,\left(a+b\,\text{Csc}[c+d\,x]^2\right)^3\,\text{Sin}[c+d\,x]^5}{15\,d\,\left(-a-2\,b+a\,\text{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^3}-\frac{8\,a^3\,\left(c+d\,x\right)\,\left(a+b\,\text{Csc}[c+d\,x]^2\right)^3\,\text{Sin}[c+d\,x]^6}{d\,\left(-a-2\,b+a\,\text{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^3}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\,\text{Csc}[c+d\,x]^2\right)^2\,dx$$

Optimal (type 3, 41 leaves, 4 steps):

$$a^2\,x-\frac{b\,\left(2\,a+b\right)\,\text{Cot}[c+d\,x]}{d}-\frac{b^2\,\text{Cot}[c+d\,x]^3}{3\,d}$$

Result (type 3, 83 leaves):

$$-\frac{4\,\left(a+b\,\text{Csc}[c+d\,x]^2\right)^2\,\left(-3\,a^2\,\left(c+d\,x\right)+b\,\text{Cot}[c+d\,x]\,\left(6\,a+2\,b+b\,\text{Csc}[c+d\,x]^2\right)\right)\,\text{Sin}[c+d\,x]^4}{3\,d\,\left(a+2\,b-a\,\text{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^2}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\,\text{Csc}[c+d\,x]^2\right)^4}\,dx$$

Optimal (type 3, 204 leaves, 7 steps):

$$\frac{x}{a^4}+\frac{\sqrt{b}\,\left(35 a^3+70 a^2 b+56 a b^2+16 b^3\right) \text{ArcTan}\left[\frac{\sqrt{b}\,\text{Cot}[c+d x]}{\sqrt{a+b}}\right]}{16 a^4 (a+b)^{7/2} d}+$$

$$\frac{b \,\text{Cot}[c+d x]}{6 a (a+b) d \left(a+b+b \,\text{Cot}[c+d x]^2\right)^3}+\frac{b (11 a+6 b) \,\text{Cot}[c+d x]}{24 a^2 (a+b)^2 d \left(a+b+b \,\text{Cot}[c+d x]^2\right)^2}+\frac{b \left(19 a^2+22 a b+8 b^2\right) \text{Cot}[c+d x]}{16 a^3 (a+b)^3 d \left(a+b+b \,\text{Cot}[c+d x]^2\right)}$$

Result (type 3, 410 leaves):

$$\frac{(c+d x) \left(-a-2 b+a \,\text{Cos}[2 (c+d x)]\right)^4 \text{Csc}[c+d x]^8}{16 a^4 d \left(a+b \,\text{Csc}[c+d x]^2\right)^4}-$$

$$\frac{\sqrt{b}\,\left(35 a^3+70 a^2 b+56 a b^2+16 b^3\right) \text{ArcTan}\left[\frac{\sqrt{a+b}\,\text{Tan}[c+d x]}{\sqrt{b}}\right] \left(-a-2 b+a \,\text{Cos}[2 (c+d x)]\right)^4 \text{Csc}[c+d x]^8}{256 a^4 (a+b)^{7/2} d \left(a+b \,\text{Csc}[c+d x]^2\right)^4}-$$

$$\frac{b^3 \left(-a-2 b+a \,\text{Cos}[2 (c+d x)]\right) \text{Csc}[c+d x]^8 \,\text{Sin}[2 (c+d x)]}{24 a^3 (a+b) d \left(a+b \,\text{Csc}[c+d x]^2\right)^4}+$$

$$\left(\left(-a-2 b+a \,\text{Cos}[2 (c+d x)]\right)^3 \text{Csc}[c+d x]^8 \left(-87 a^2 b \,\text{Sin}[2 (c+d x)]-116 a b^2 \,\text{Sin}[2 (c+d x)]-44 b^3 \,\text{Sin}[2 (c+d x)]\right)\right) \Big/$$

$$\left(768 a^3 (a+b)^3 d \left(a+b \,\text{Csc}[c+d x]^2\right)^4\right)+\frac{\left(-a-2 b+a \,\text{Cos}[2 (c+d x)]\right)^2 \text{Csc}[c+d x]^8 \left(-19 a b^2 \,\text{Sin}[2 (c+d x)]-14 b^3 \,\text{Sin}[2 (c+d x)]\right)}{192 a^3 (a+b)^2 d \left(a+b \,\text{Csc}[c+d x]^2\right)^4}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \left(a+b \,\text{Csc}[c+d x]^2\right)^{5/2} dx$$

Optimal (type 3, 167 leaves, 8 steps):

$$-\frac{a^{5/2} \,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Cot}[c+d x]}{\sqrt{a+b+b \,\text{Cot}[c+d x]^2}}\right]}{d}-\frac{\sqrt{b}\,\left(15 a^2+10 a b+3 b^2\right) \text{ArcTanh}\left[\frac{\sqrt{b}\,\text{Cot}[c+d x]}{\sqrt{a+b+b \,\text{Cot}[c+d x]^2}}\right]}{8 d}-$$

$$\frac{b (7 a+3 b) \,\text{Cot}[c+d x] \sqrt{a+b+b \,\text{Cot}[c+d x]^2}}{8 d}-\frac{b \,\text{Cot}[c+d x] \left(a+b+b \,\text{Cot}[c+d x]^2\right)^{3/2}}{4 d}$$

Result (type 3, 396 leaves):

$$-\frac{\left(-4 a^3-15 a^2 b-10 a b^2-3 b^3\right) \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{-b} \text{Cos}[c+d x]}{\sqrt{-a-2 b-a \text{Cos}\left[2 \left(-c+\frac{\pi}{2}-d x\right)\right]}}\right] \left(a+b \text{Csc}[c+d x]^2\right)^{5/2} \text{Sin}[c+d x]^5}{\sqrt{2} \sqrt{-b}\, d \left(-a-2 b+a \text{Cos}[2 (c+d x)]\right)^{5/2}}+$$

$$\left(\left(a+b \text{Csc}[c+d x]^2\right)^{5/2} \left(-\frac{3}{2} \left(3 a b \text{Cos}[c+d x]+b^2 \text{Cos}[c+d x]\right) \text{Csc}[c+d x]^2-b^2 \text{Cot}[c+d x] \text{Csc}[c+d x]^3\right) \text{Sin}[c+d x]^5\right) \Big/ \left(d \left(-a-2 b+a \text{Cos}[2 (c+d x)]\right)^2\right)+$$

$$\left(4 a^3 \left(a+b \text{Csc}[c+d x]^2\right)^{5/2} \left(-\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{-b} \text{Cos}[c+d x]}{\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]}}\right]}{\sqrt{2} \sqrt{-b}}+\frac{\sqrt{2} \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cos}[c+d x]+\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]}\right]}{\sqrt{a}}\right) \text{Sin}[c+d x]^5\right) \Big/ \left(d \left(-a-2 b+a \text{Cos}[2 (c+d x)]\right)^{5/2}\right)$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b \text{Csc}[c+d x]^2}} \, dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Cot}[c+d x]}{\sqrt{a+b \text{Csc}[c+d x]^2}}\right]}{\sqrt{a}\, d}$$

Result (type 3, 98 leaves):

$$-\frac{\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]} \text{Csc}[c+d x] \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cos}[c+d x]+\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]}\right]}{\sqrt{2} \sqrt{a}\, d \sqrt{a+b \text{Csc}[c+d x]^2}}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \left(1+\text{Csc}[x]^2\right)^{3/2} \, dx$$

Optimal (type 3, 44 leaves, 6 steps):

$$-2 \text{ArcSinh}\left[\frac{\text{Cot}[x]}{\sqrt{2}}\right]-\text{ArcTan}\left[\frac{\text{Cot}[x]}{\sqrt{2+\text{Cot}[x]^2}}\right]-\frac{1}{2} \text{Cot}[x] \sqrt{2+\text{Cot}[x]^2}$$

Result (type 3, 94 leaves):

$$\frac{1}{(-3+\text{Cos}[2x])^{3/2}}(1+\text{Csc}[x]^2)^{3/2}\left(-4\sqrt{2}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\text{Cos}[x]}{\sqrt{-3+\text{Cos}[2x]}}\right]+\sqrt{-3+\text{Cos}[2x]}\,\text{Cot}[x]\,\text{Csc}[x]-2\sqrt{2}\,\text{Log}\left[\sqrt{2}\,\text{Cos}[x]+\sqrt{-3+\text{Cos}[2x]}\right]\right)\text{Sin}[x]^3$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int\sqrt{1+\text{Csc}[x]^2}\,dx$$

Optimal (type 3, 28 leaves, 5 steps):

$$-\text{ArcSinh}\left[\frac{\text{Cot}[x]}{\sqrt{2}}\right]-\text{ArcTan}\left[\frac{\text{Cot}[x]}{\sqrt{2+\text{Cot}[x]^2}}\right]$$

Result (type 3, 68 leaves):

$$\frac{\sqrt{2}\sqrt{1+\text{Csc}[x]^2}\left(\text{ArcTan}\left[\frac{\sqrt{2}\,\text{Cos}[x]}{\sqrt{-3+\text{Cos}[2x]}}\right]+\text{Log}\left[\sqrt{2}\,\text{Cos}[x]+\sqrt{-3+\text{Cos}[2x]}\right]\right)\text{Sin}[x]}{\sqrt{-3+\text{Cos}[2x]}}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{1+\text{Csc}[x]^2}}\,dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$-\text{ArcTan}\left[\frac{\text{Cot}[x]}{\sqrt{2+\text{Cot}[x]^2}}\right]$$

Result (type 3, 49 leaves):

$$-\frac{\sqrt{-3+\text{Cos}[2x]}\,\text{Csc}[x]\,\text{Log}\left[\sqrt{2}\,\text{Cos}[x]+\sqrt{-3+\text{Cos}[2x]}\right]}{\sqrt{2}\sqrt{1+\text{Csc}[x]^2}}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int\sqrt{-1-\text{Csc}[x]^2}\,dx$$

Optimal (type 3, 33 leaves, 6 steps):

$$\text{ArcTan}\left[\frac{\text{Cot}[x]}{\sqrt{-2-\text{Cot}[x]^2}}\right]+\text{ArcTanh}\left[\frac{\text{Cot}[x]}{\sqrt{-2-\text{Cot}[x]^2}}\right]$$

Result (type 3, 70 leaves):

$$\frac{\sqrt{2}\ \sqrt{-1-\text{Csc}[x]^2}\ \left(\text{ArcTan}\left[\frac{\sqrt{2}\ \text{Cos}[x]}{\sqrt{-3+\text{Cos}[2\,x]}}\right]+\text{Log}\left[\sqrt{2}\ \text{Cos}[x]+\sqrt{-3+\text{Cos}[2\,x]}\right]\right)\ \text{Sin}[x]}{\sqrt{-3+\text{Cos}[2\,x]}}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1-\text{Csc}[x]^2}}\,dx$$

Optimal (type 3, 18 leaves, 3 steps):

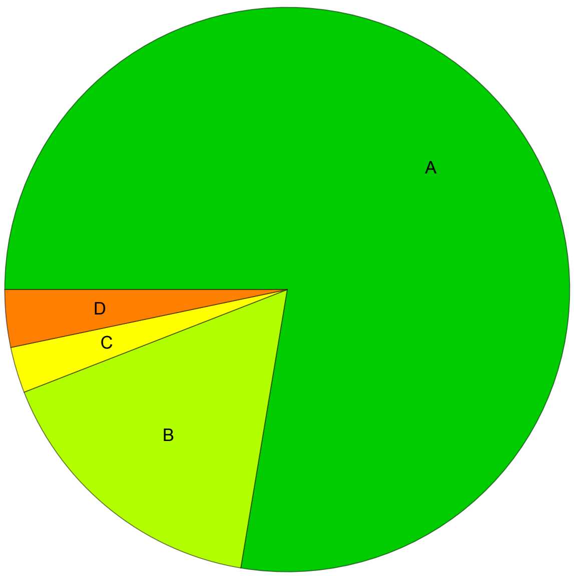
$$-\text{ArcTanh}\left[\frac{\text{Cot}[x]}{\sqrt{-2-\text{Cot}[x]^2}}\right]$$

Result (type 3, 51 leaves):

$$-\frac{\sqrt{-3+\text{Cos}[2\,x]}\ \text{Csc}[x]\ \text{Log}\left[\sqrt{2}\ \text{Cos}[x]+\sqrt{-3+\text{Cos}[2\,x]}\right]}{\sqrt{2}\ \sqrt{-1-\text{Csc}[x]^2}}$$

Summary of Integration Test Results

304 integration problems



A - 236 optimal antiderivatives

B - 50 more than twice size of optimal antiderivatives

C - 8 unnecessarily complex antiderivatives

D - 10 unable to integrate problems

E - 0 integration timeouts